Inverse-Turbulent Prandtl Number Effects on Reynolds Numbers of RNG k-ε Turbulence Model on Cylindrical-Curved Pipe

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Inverse-turbulent Prandtl number effects on Reynolds numbers of RNG *k-ε* turbulence model on cylindrical-curved pipe

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Abstract. Inverse-turbulent Prandtl number (α) is one of important parameters on RNG *k-ε* turbulence model which represent the cascade energy of the flow, which occur in cylindrical curved-pipe. Although many research has been done, turbulent flow in curved pipe is still a challanging problem. The range of α of the basic RNG *k-ε* turbulence model described by Yakhot and Orszag (1986) with range 1-1.3929 has to be more specific on Reynolds number (Re) and geometry. However, since the viscosity is sensitive to velocity and temperature, the specific value of α is needed on specific range of Re. This paper is aimed to gain optimum α of the flow in curved pipe with upper and lower Re which simulated numerically with CFD. The Re at the inlet side were; Re = 13000 and Re = 63800 on cylindrical curved-pipe with r/D of 1.607.The α were varied to 1, 1.1, 1.2, 1.3. The curved pipe was an cylindrical air pipe with 43mm inlet diameter. The computational grid that is used for CFD numerical simulation with CFDSOF®, hexagonal-surface fitted consist of 139440 cells. CFD simulation done with α varies by 1, 1.1, 1.2, dan 1.3. The wall is assumed to zero-roughness. The CFD simulation generated several results; at Re 13000, the value of α did not affect the turbulent parameter which also confirmed the basic therory of RNG *k-ε* turbulence model that the minimum Re of α *is* 2.5 x 10^4. At Re = 63800, the use of α of 1.1 shows more turbulent flow domination on molecular flow. Lower eddy dissipation by 1.67%, increasing turbulent kinetic energy by 2.2%, and Effective viscosity increase by 4.7% compared to α = 1. Therefore, the use of α 1.1 is the most suitable value to be used to represent turbulent flow in curved pipe with RNG *k-ε* turbulence model with Re 63800 and r/D 1.607 among others value that have discussed in this paper.

Nomenclature

c_v	: proportional k constant (0.09)	α_0	: Inverse Prandtl number (molecular)
C_k	: Kolmogorov constant (1.3-2.3)	α	: Inverse-Turbulent Prandtl number
$E(k)$	: velocity fluctuation spectrum (m^2/s)	ε	: turbulent dissipation (m^2/s^3)
f	: random force by velocity spectrum	k	: dissipation range
K	: turbulent kinetic energy (m^2/s^2)	ν_{eff}	: total viscosity
P_t	: Turbulent Prandtl number	ν	: turbulent viscosity (m^2/s)
v	: velocity (m/s)	ν_0	: molecular viscosity $(kg/m.s)$
ρ	: density (kg/m^3)	ν_T	: eddy viscosity $(kg/m.s)$
p	: pressure (N/m^2)	$u'_i u'_j$	: Reynolds stress

Introduction

It is already known that turbulent flow is a very unique and specific flow. In order to get analyzed, the flow can be modeled numerically by turbulence model. Turbulence model is increasing greately in the past decades and has became more specific along with the need of more detail flow analysis. A lot of parameters involves in turbulence model, constants and value with specified and unspecified range. Among the RANS-based turbulence model, RNG *k-ε* is a promising turbulence model to predict swirling flow and scondary fow that attached [1-5]. Since turbulent flow also occur along with energy cascade with viscosity stratafication, turbulent Prandtl number (Pr_t) is one of important parameters which relates molecular viscosity and eddy viscosity. Those flow parameters undoubtly occur in curved-pipe. As has investigated by Noorani et.al [6], flow phenomena on turbulent flow in straight to curved pipe, flow at outside curve has much smaller eddy than the mean condition thus flow with higher local Reynolds number along with higher eddy viscosity than at mean of curve [7] and lead to secondary flow to occur. This paper uses inverse-turbulent Prandtl number (α) nomenclature instead of turbulent Prandtl number ($\alpha^{-1}=Pr_t$) in order to describe the domination effect of eddy viscosity to thermal diffusivity since in this paper there are insignificant effect of critical fluid phenomena.

Alhtough many numerical research has been done for analyze; experimentaly and numerically inverse-turbulent Prandtl number, this value to be applied to RNG k-ε turbulence model has to be analyzed in more detail in order to gain better prediction of the flow phenomena. The value of turbulent Prandtl number Pr_t was initially described with Reynolds analogy (Pr_t = 1) by Kays and Crawford, which has been experimentally clarified and resulting the value of Pr_t to 0,7-0,9 [8]. Although valid for some common fluid flow, the Reynolds analogy doesn't predict satisfying result in more complex geometry and has became an issue for RANS-based turbulence model since the α has a wide range values [9].. Others research of this field also gained the range of α. Yakhot and Orszag [10] has described the range of the value of inverse-turbulent Prandtl number for general turbulent flow in RNG k-ε turbulence model which range from 1-1.3939. However very few of them gained the specific value of α, especially for the low inverse-turbulent Prandtl number cases because of its difficulty [11]. Mohseni et.al (2012) developed the correlation for the α to a super critical fluid flows with low Reynolds number k-ε turbulence model with low Re [12].

Dong et.al (2002) also has investigated the effect of Prandtl number and Reynolds number Re = 10^4 in channel flow with LES turbulence model [11]. Furthermore Ould-Rouiss et.al (2010) investigated the effect of turbulent Prandtl number on anular pipe with DNS use the turbulent Prandtl number of 0,71 and shows that this parameter has caused near-wall flow spread to the mean section and [13]. Subhas et.al presented more andvance work to develop new formula for the turbulent Prandtl number [14]. These reserach and clarification of Reynolds analogy shows that the turbulent Prandtl number is specific on RNG turbulence model. In order to get specific value of α at specific range of Reynolds number of a cylindrical-curved pipe, part of Proto X-2a Bioenergy Micro Gas Turbine that has been developed, range of sepcific α is needed. In the previous paper, the authors concluded that with that geometry (fluid = air), the optimun inverse-turbulent Prandtl number α is 1.3 at experimental data Re = 40900 with RNG k-ε turbulence model. However, since the viscosity is fllow parameters which sensitive to velocity and temperature, the reference of α is needed on lower and upper value of Reynolds number. The performance of turbulence models are highly depending on empirical parameters [12].

Therefore, flow in curved pipe simulated numerically with CFD, with RNG k-ε turbulence model. The selection of the Reynolds number was driven by the lower and the upper value, where at the lower value the compressor of the micro gas turbine is assumed to supply minumum air to the pipe, and at the upper value of Re refers to maximum air velocity at which the compressor works at its maximum condition at given constraint of input energy supplied; Re = 13000 and Re = 63800 respectively. Therefore the inverse-turbulent Prandtl number (α) were varied to 1, 1.1, 1.2, 1.3 (Pr_t = 0,7-0,9) which have been used in many numerical study. The wall is assumed to zero-roughness. This paper is aimed to gain the specific value of inverse-turbulent Prandtl number (α) in RNG k-ε turbulence model in two extreme difference Re of cylindrical-curved pipe with r/D = 1.607; Re = 13000 and Re = 63800.

Research Methodology

Governing Equations

Boussinesq Hypothesis. Momentum transfer caused by turbulent flow can be modeled by the use of eddy viscosity. Molecular viscosity and eddy viscosity occured in flow caused the Reynolds stress. The Reynolds stress can be defined as velocity gradient [15], [16], [17]. Therefore, total viscosity of the flow became: [2].

$$\rho\overline{u_i' u_j'} = \frac{2}{3}\rho k\delta_{ij} + \left(v_t\left[\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i}\right]\right) \quad (1)$$

$$v_{eff} = v_T + v \quad (2)$$

$$v_T = c_v \frac{K^2}{\varepsilon} \quad (3)$$

Kolmogorov Theorem. This theorem described how energy is transfered from large scale eddies to small scale eddies, and how the energy is dissipated [1], [2], [10]. In turbulent flow, amount of energy supplied can be assumed equal to be dissipated on certain rate. In smale scale eddies, this change of energy related to the flow time scale generally. Therefore, this energy rate is small compared to energy dissipated [18]. In Kolmogorov theorem, velocity fluctuations tends to be universal, where the amount of energy assumed only depends on turbulent dissipation rate (ε) and length scale (l) [10], [19].

$$E(k) = C_k \varepsilon^{-2/3} k^{-5/3} \quad (4)$$

Navier-Stokes Equation. In Newtonian fluid, flow acceleration and forces acompany related with Navier-Stokes equation [20]. The RNG k-ε turbulence model uses the normalized Navier-Stokes equation:

$$\frac{\partial \mathrm{v}}{\partial t} = (\mathrm{v}.\nabla)\mathrm{v} = f - \frac{1}{\rho}\nabla p + v_0 \nabla^2 \mathrm{v} \quad (5)$$

Inverse-Turbulent Prandtl Number. Inverse-Turbulent Prandtl number, α described the ratio between thermal diffusivity and turbulent viscosity:

$$\alpha \equiv Pt^{-1} = \frac{X}{v_T} \quad (6)$$

Generally, with temperature transport on horizontal way assumed only to be affected by temperature fluctuation by temperature gradient, value of α assume equal to one [18]. Furthermore, the range of α 1.1-1.3929 also can be considered to flow analysis [10]:

$$\left|\frac{\alpha - 1.3929}{\alpha_0 - 1.3929}\right|^{0.6321} \left|\frac{\alpha + 2.3929}{\alpha_0 + 2.3929}\right|^{0.3679} = \frac{v_0}{v_T} \quad (7)$$

In general fluids, the heat transfer is dominated by molecular diffusion, the thermal resistance is distributed over the entire cross-section, and the turbulent Prandtl number, α, assumed one [21].

Transport Equations. RNG k-ε turbulence models is a RANS-based turbulence model with two transport equation; k (turbulence kinetic energy) and ε (turbulent dissipation).

Transport equation of k [10], [2]

$$\frac{\partial K}{\partial t} + (\overline{\mathrm{v}}.\nabla)K = \frac{v_T}{2}\left(\frac{\partial \overline{v}_i}{\partial x_j} + \frac{\partial \overline{v_j}}{\partial x_i}\right)^2 - \overline{\varepsilon} + \frac{\partial}{\partial x_i}\alpha_K v_T \frac{\partial K}{\partial x_i} \quad (8)$$

Transport equation of ε [10]

$$\frac{D\overline{\varepsilon}}{Dt} = \frac{v}{2}\left(\frac{\partial \overline{v}_i}{\partial x_j} + \frac{\partial \overline{v_j}}{\partial x_i}\right)^2 - 1.7215\frac{\overline{\varepsilon}^2}{K} + \frac{\partial}{\partial x_i}\alpha_\varepsilon v \frac{\partial \overline{\varepsilon}}{\partial x_i} \quad (9)$$

Geometrical Model And Experimental Set-Up

Pipe geometry and CFD model of the cylinderical durved-pipe are represented on Fig. 1.

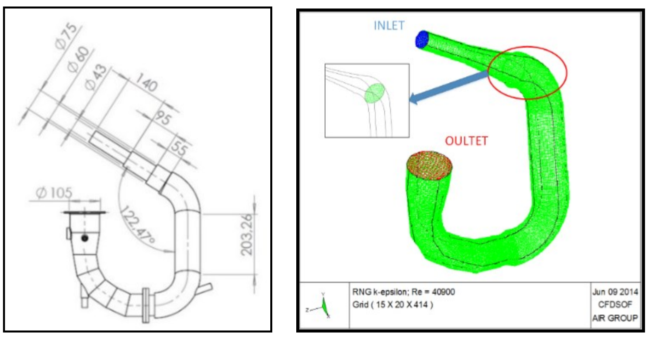

Fig 1. (a) Pipe geometry model and (b) CFD model

CFD Model. The computational grid that is used for CFD numerical simulation with CFDSOF® is shown Fig. 1. The grid was hexagonal-surface fitted 139440 cells. CFD simulation done with inlet Reynolds number of 13000 and 63800 and turbulent Prandtl number α varies by 1, 1.1, 1.2, dan 1.3. The von-Karman constant is assumed to 0.41, zero-roughness wall.

Results and Discussion

Grid dependency of the curved-pipe geometry and numerical validation has done to avoid numerical error. The result from several turbulent flow shown on figs 2-10. The results from α compared to the result from α =1 at which the molecular viscosity is assumed equal to turbulent viscosity. The predicted of velocity magnitude is shown on figs 2 and 3 with Re = 13000 and 63800 respectively. The variation of α does not predict to much different velocity magnitude profile except by their velocity. This is because figure1 and 2 show the velocity magnitude at which the vector resultant of velocity. The inclination of pipe is the main reason why the velocity profile is represented by velocity magnitude. Generally, for both Re 13000 and Re 63800, the velocity profile on cross section at curved show secondary flows. CFD simulation at Re = 13000 shows there is no significant difference between all varied inverse-turbulent Prandtl number (α). Furthermore, similiar velocity profile also resulted at Re = 63800, although slightly different prediction shown with α = 1.1 at which velocity profile was different from the others varied α. This result is surprising since the velocity profile was a resultant from velocity component. At Re = 13000, Eddy dissipation on fig 4 shows similiar result for all varried inverse-turbulent Prandtl number (α). The eddy dissipation with maximum value of 2.17E-02 m^2/s^3 concentrated at zone at which secondary flows occur. This condition confirmed equation (3) relationship with turbulent kinetic energy which shown on fig 6 where maximum kinetic energy transfered at whit dissipation occur.

Difference prediction shows of eddy dissipation at Re = 63800 in fig 5. While the use of α = 1.2, and 1.3 shows the same maximum value of eddy dissipation of 4.67 m^2/s^3 which similiar to α 1, the use of α = 1.1 is about 1.67% lower. This condition also can be seen in velocity magnitude profile where the use of α = 1.1 has increased maximum velocity at outer side about 1.1%. With higher prediction of velocity, dissipation less occur, pararel with turbulent kinetic energy where 2.2% more kinetic energy transfered et α = 1.1. Figure 9 show the efffective viscosity at Re = 63800. Similiar to figs 3,5, and 7, the effective viscosity with α = 1.1 shows different trends with others varied α. Instead of similiar to α 1, 1.2, and 1.3 which predicted to had similiar maximum value 10.6 kg/m-s, the use of α 1.1 shows increasing effective viscosity of 4.7% than others. This is beacuse according to equation (3), the increasing of turbulent kinetic energy by 2.2% as shown on fig 7 and the decrasing of eddy dissipation by 1.67% as shown on fig 5 really affected the effective viscosity. Furthermore, the increrasing effective viscosity shows that turbulent flow captured more clearly than other values of varied α. It is clear that at Re = 13000, the varied α did not affect the prediction results on all turbulent flow parameters. This result is consistent with relationship between molecular viscosity and turbulent viscosity in equation (7) which proposed by Yakhot and Orszag et.al [22] valid for $2.5 \times 10^4 < Re < 10^6$. This also explained that at Re = 13000, turbulent flow is much less dominant than the molecular flow.

According to transport equation of k in figure (8), the increasing value of effective viscosity clearly shows also increasing the production and diffusion terms and the decreasing of eddy dissipation also decrease the dissipation on k transport equation and on the ε transport equation as well. In generall, according to the result with Re = 63800 on α 1, 1.1, 1.2, and 1.3, the use of α 1.1 shows that turbulent viscosity is more dominant than molecular viscosity and therefore shows the viscosity stratafication more fairly. This value is close with α described by Kays et.al at which found α of 1,16 in circular tube [8]. According to equation (7) shows with α 1.2 and 1.3, the turbulent viscosity is too dominant to molecular viscosity. Furthermore, based on the characetristics if turbulent flow and turbulent flow parameters; increasing turbulent kinetic energy and effective viscosity, α 1.1 represent the turbulent flow at the curved-pipe better than other values of α.

Velocity Magnitude

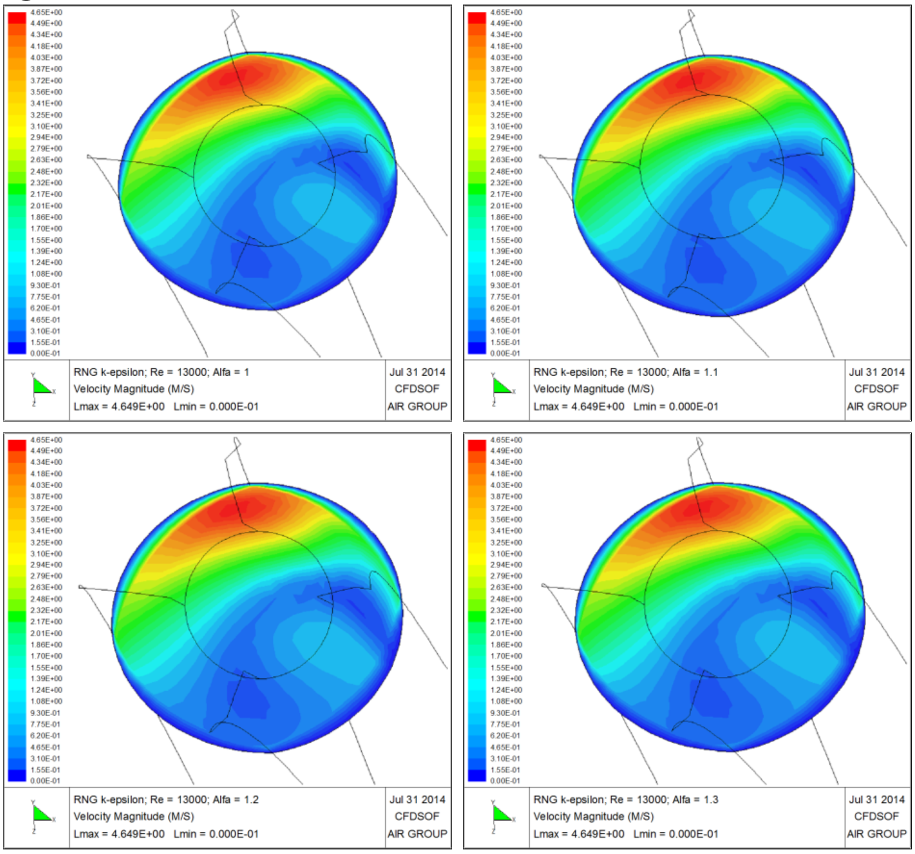

Fig 2. Velocity Magnitude Re = 13000

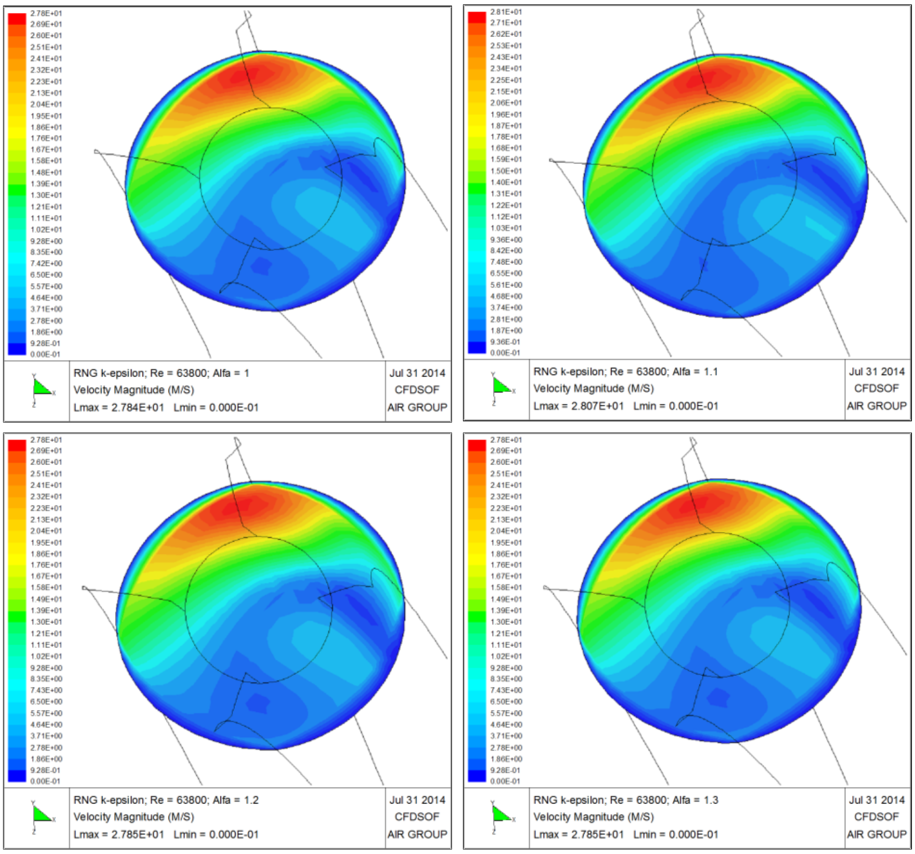

Fig 3. Velocity Magnitude; Re = 63800

Eddy dissipation

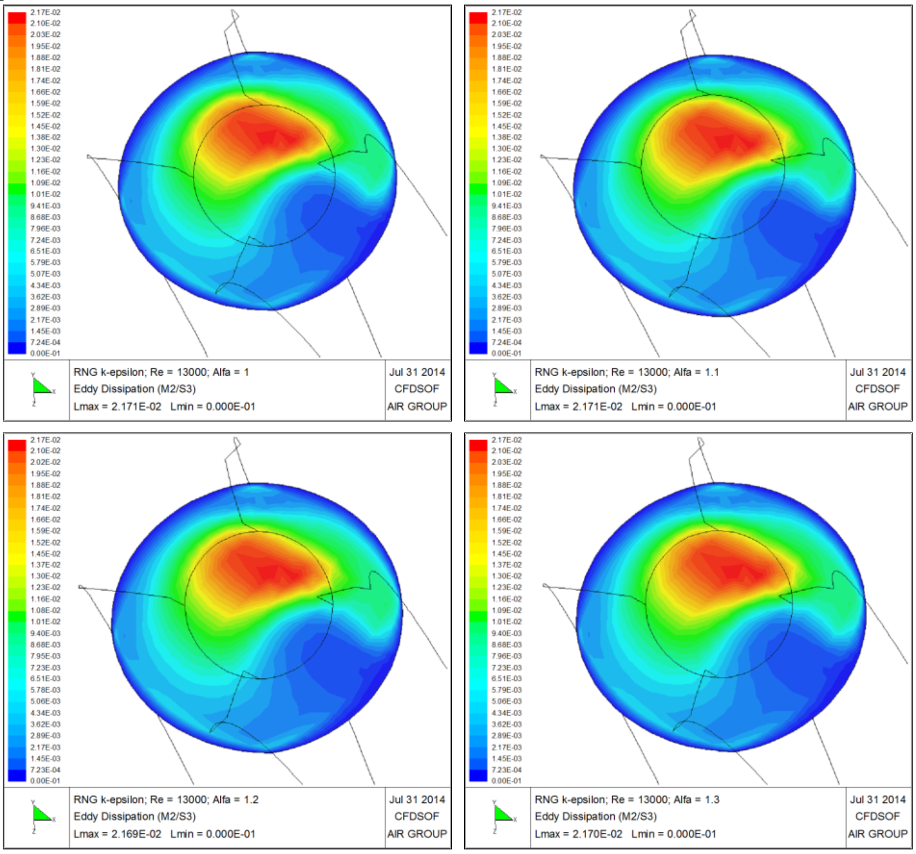

Fig 4. Eddy Dissipation; Re = 13000

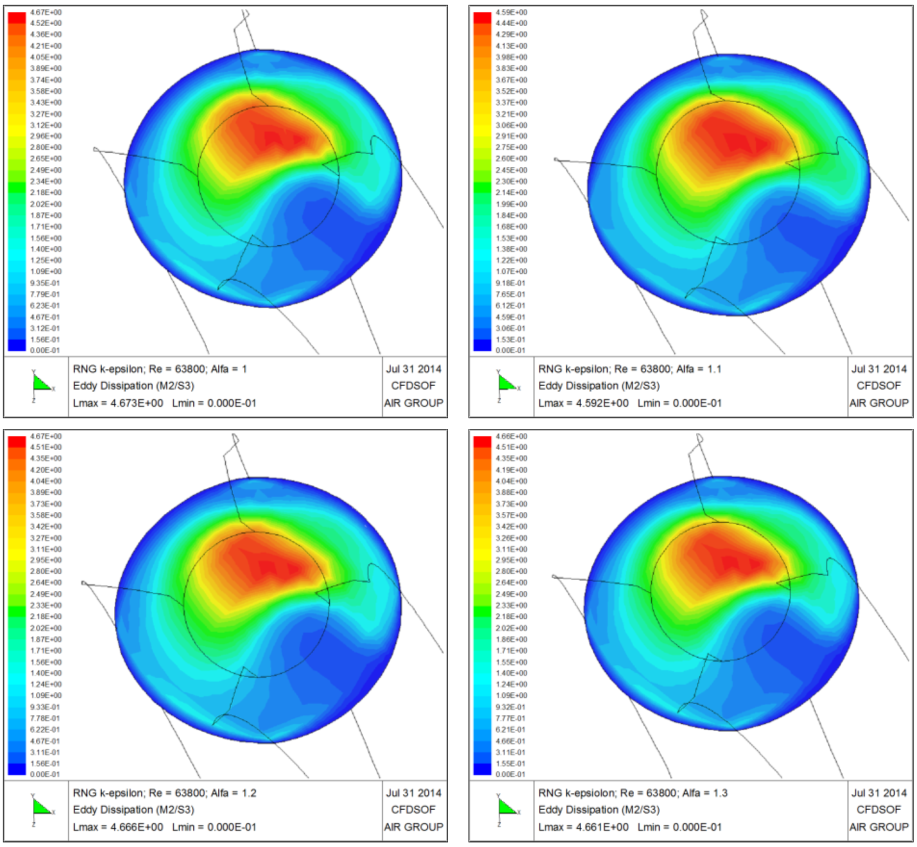

Fig 5. Eddy Dissipation; Re = 63800

Turbulent Kinetic Energy

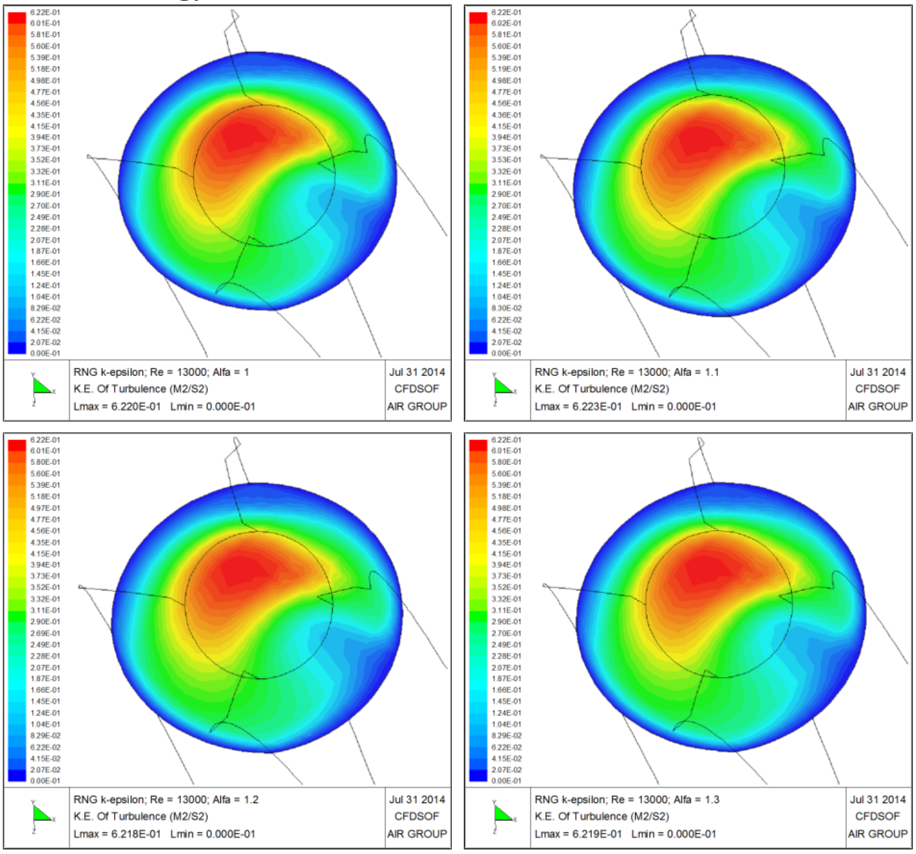

Fig 6. Turbulent Kinetic Energy; Re = 13000

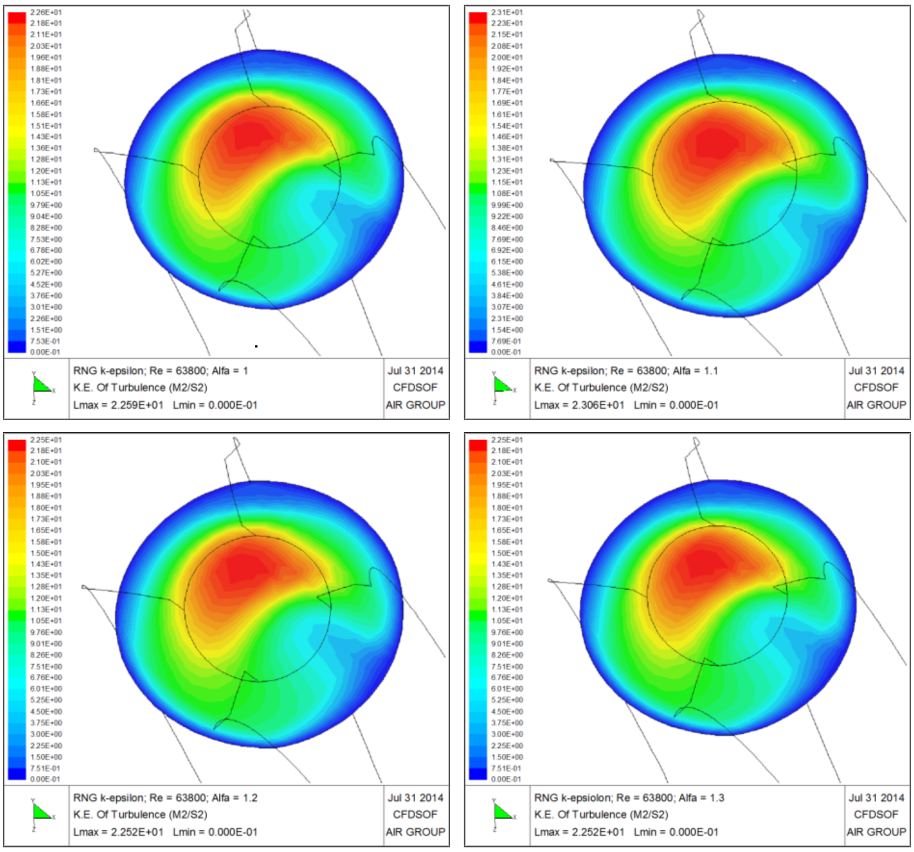

Fig 7. Turbulent Kinetic Energy; Re = 63800

Effective Viscosity

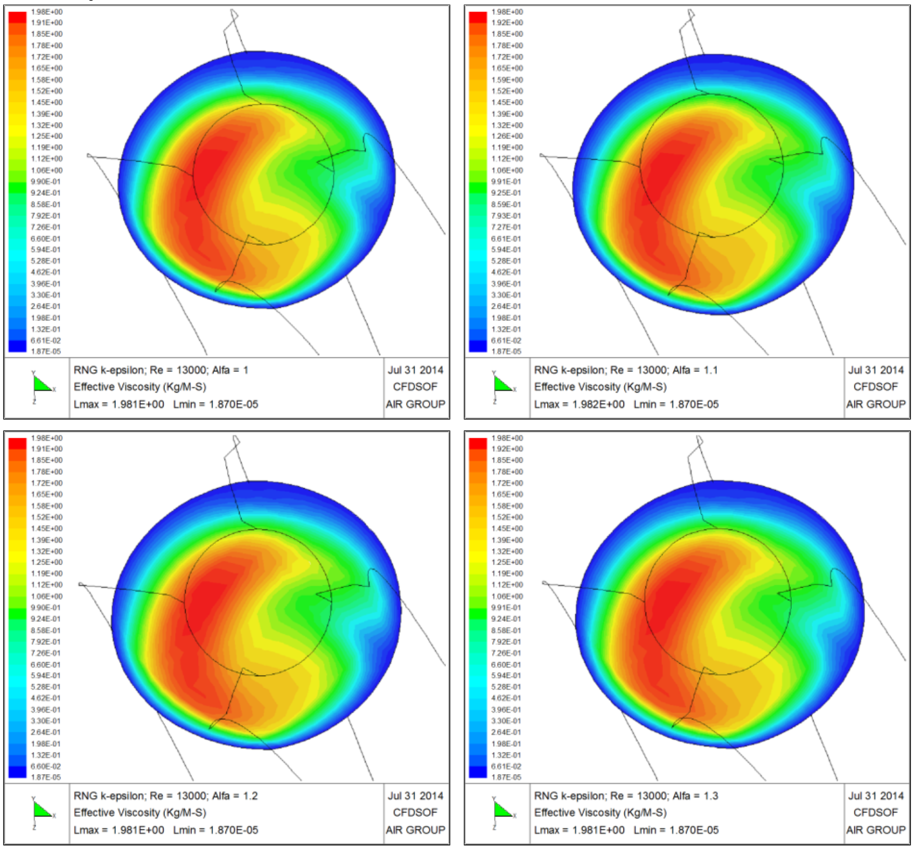

Fig 8. Effective Viscosity; Re = 13000

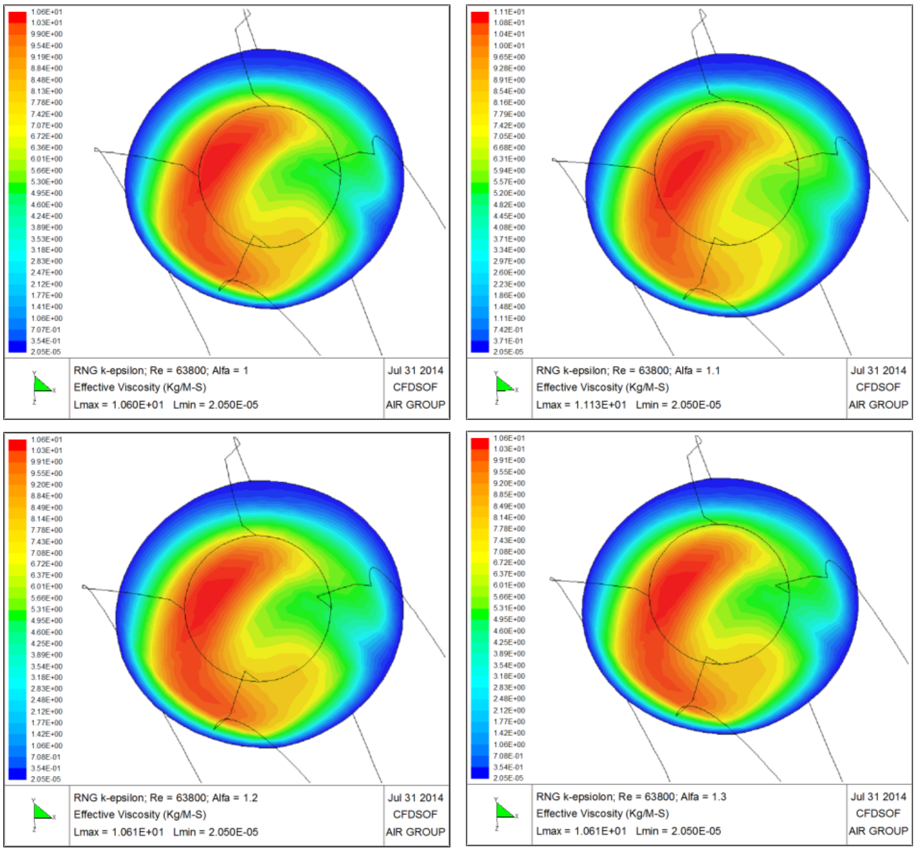

Fig 9. Effective Viscosity; Re = 63800

Conclusions

CFD simulation with RNG k-ε has been performed. The result revealed are follows; at Re = 13000, the variation of inverse-turbulent Prandtl number, α 1, 1.1, 1.2, and 1.3 shows insginificant result. This indicated that at this Re, turbulent flow occur in curved-pipe is less-dominant than the molecular flow. This result also confirmed the theory proposed by Yakhot & Orszag that the minimum value of Re that α can affect the predicted flow is 2.5 x 10^4. Further more, at Re = 63800, the use of inverse-turbulent Prandtl number α of 1.1 shows more turbulent flow domination on molecular flow. This condition indicated by several turublent flow parameters; lower eddy dissipation by 1.67%, increasing turbulent kinetic energy by 2.2%, and Effective viscosity increase by 4.7% compared to α = 1. Therefore, the use of α 1.1 is the most suitable value to be used to represent turbulent flow in curved pipe with r/D of 1.607 with RNG k-ε turbulence model among others value that has been discussed in this paper.

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